Buckling characteristics of a circular toroidal shell with stiffened ribs

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Abstract

The ring-stiffened toroidal shell is entirely different from the traditional ring-stiffened cylindrical and spherical shells. It has special structurally beneficial characteristics for underwater engineering. A whole welding steel toroidal model with the ring-stiffened ribs has been manufactured, tested until collapse in pressure chamber and analyzed by nonlinear finite element analyses (FEAs). Then the essential cause and collapse mode of this tested ring-stiffened toroidal model have been studied comprehensively by nonlinear FE method. The experimental and numerical results both reveal that the initial geometric imperfection would mainly determine the model’s final failure mode, critical pressure and collapse deformation. Further parametric study is made to study the buckling property of ring-stiffened toroidal shell, including failure mode and critical pressure loading by varying the structural parameters. This investigation will lay a good foundation for deriving a theoretical solution for the buckling of ring-stiffened toroidal shell in the near future.

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Keywords:
Toroidal shell
Steel model
Experiment
Nonlinear FEM
Failure mode

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1. Introduction

Although it is more difficult for analysis, design and manufacture in comparing with the cylindrical, spherical or conical shells, toroidal shell is still widely used as joint components or accessories in different engineering fields for its specific benefits, including pressure vessel and piping industry, nuclear power industry and ocean engineering. Especially it should be noticed that by considering the advantages and disadvantages of using a circular cylindrical or spherical shell as pressure hull, Ross (2005, 2006) recently had proposed a new conceptual design of an underwater vehicle and an underwater space station with the main pressure hull in such a toroidal shape.

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0029-8018/© 2015 Elsevier Ltd. All rights reserved.
The pure toroidal shell is a special shell of revolution for the existence of horizontal tangents at \( \varphi = \pm \varphi_0/2 \), which are called turning points. The Gaussian curvature ratio changes its sign from positive to negative (see Fig. 1, where \( R \) represents the distance of the center of the meridian circles to the axis of rotation, \( \varphi \) is the tangential angle of the shell, \( r \) is the curvature radius of the parallel circle on the toroidal shell to revolving direction, \( t \) is the wall thickness of the shell, \( a \) is the radius of the meridian circle, \( \theta_0 \) is the rotational angle of the meridian circle). When the toroidal shell has ring-stiffened ribs uniformly in toroidal direction, its revolution property would be disappeared and it will just leave one symmetrical plane \( xOy \). However if the ring-stiffened ribs are set evenly on the toroidal model, the planes of ribs and medial planes between two arbitrary adjacent ribs are also symmetrical planes. Here \( 2\theta_0 \) is the rotational angle of the meridian circle between two arbitrary adjacent ribs.

Because of its potential application in ocean engineering and the difficulty in deriving a theoretical solution, several researchers have spent great efforts to analyze such a type of toroidal shells, whose goal is to derive its theoretical solution and even to obtain its simple analysis and design method for engineering applications. It can be found that all of previous works on toroidal shells have almost focused on pure toroidal shells which have no stiffened ribs. Flügge and Sobel (1965) and Sobel and Flügge (1967) had carried out the buckling and stability analysis of toroidal shells. Bushnell (1967), Jordan (1973) and Panagiotopoulos (1985) had carried out the stability analysis of the toroidal shells by finite difference method (FDM). The buckling of the segments of toroidal shells had been solved by Stein and McElman (1965) and Hutchinson (1967), Galletly and Blachut (1995); Galletly (1998) had even studied the stability of closed toroidal shells with circular or non-circular cross-sections. Blachut and Jaiswal (2000), Blachut (2003), Blachut and Smith (2008) and Blachut (2014) have studied the buckling of toroidal shells as segment or entirety by FEM and model tests. Wang (1989) had carried out the analysis of geometric nonlinear buckling and post-buckling of toroidal shells by asymptotic approach. Based on a shell-theory FEM, Zhan and Redekop (2008) had even obtained analysis of natural frequencies, buckling loads and collapse pressures of toroidal ranks with ovaloid cross-section. Recently Zingoni (2015) reviewed and emphasized the importance and application of toroidal shell.

All of above researchers had tried to solve the pure toroidal shell or its segments by elastic or nonlinear analyses. Du et al. (2010) had shown some nonlinear structural characteristics of a segment of ring-stiffened toroidal shell with certain parameters by FEM and it was confirmed that such a type of shells could be used as main pressure hull in underwater engineering similar as ring-stiffened cylinder. So based on the curve-beam theory with elastic supporting and assumption of symmetry on two specially solved cycle lines, Zou et al. (2012) provided a theoretical method to solve the special strength of ring-stiffened toroidal shell on internal and external toroidal cycle positions. Du et al. (2014), according to the curve-beam theory and exact displacement–strain relation, provided a simplified theoretical solution of ring-stiffened toroidal shell for arbitrary position and verified the strength solution on some of its key positions by static elastic model experiment under external pressure.

In this paper a whole welding steel model of toroidal shell with ring-stiffened ribs is manufactured and tested in hydrostatic pressure chamber. Its failure mode, ultimate collapsed deformation and critical pressure will be totally obtained by experimental method. By the nonlinear FE method, the tested toroidal model is studied to explore structural characteristics. The natural failure modes of tested toroidal model under external pressure will be discussed. And finally the experimental and numerical results confirm that the initial geometric imperfection would affect and determine the model ultimate collapsing shape.

### 2. Model experiment of the toroidal shell with ring-stiffened ribs

Experimental method is an effective means to demonstrate and reveal the structural performance or verify the theoretical and numerical methods. Therefore a steel welding model was made and test was carried out. The objective of the model test is focused on obtaining its critical pressure loading and showing its ultimate collapse shape.

#### 2.1. Design and manufacture of the toroidal model with ring-stiffened ribs

The complete toroidal shell will lose its symmetrical characteristic after its ring-stiffened ribs setting up, which obviously will add more difficulty for the theoretical analysis. Moreover, to increase the ability of the toroidal model with ring-stiffened ribs

---

**Table 1**

<table>
<thead>
<tr>
<th>( R ) (mm)</th>
<th>( a ) (mm)</th>
<th>( t(c_0) ) (mm)</th>
<th>( t(0) ) (mm)</th>
<th>( E ) (MPa)</th>
<th>( \mu )</th>
<th>( \sigma_s ) (MPa)</th>
<th>( \sigma_f ) (MPa)</th>
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</thead>
<tbody>
<tr>
<td>381</td>
<td>131.5</td>
<td>10(12)</td>
<td>10 × 40</td>
<td>2.01 × 10^5</td>
<td>0.3</td>
<td>325</td>
<td>485</td>
</tr>
</tbody>
</table>

![Fig. 1. Geometry model and parameters of the toroidal shell with ring-stiffened ribs.](image-url)
To resist external pressure, the ring-stiffened ribs should be installed uniformly in the toroidal direction and each rib must be set in radial direction of the toroidal shell and kept in one plane which should pass the axle center of the toroidal model.

Based on the geometric property of the toroidal shell with stiffened ribs and the pressure equipment limit, the toroidal model was designed to have a collapse pressure lower than 70 MPa which is the maximum pressure allowed by the pressure chamber. Its dimensions and material properties are shown in Table 1. The model graphics is shown in Fig. 2.

To manufacture the model of the toroidal shell with ring-stiffened ribs, there are three steps to fabricate the welded steel toroidal model. Four elbow pipes (A, B, C and D) are firstly molded into four parts which can be joined to a complete toroidal pipe, shown in Fig. 3. Nominal thickness of one elbow (elbow A in Fig. 2) is 12 mm while the nominal thickness for all others (elbows B, C, D in Fig. 2) is 10 mm, because thick elbow A needs to open for setting watertight connectors. The second step is to measure and check the thickness of elbow pipes shell and their shape dimensions including toroidal radius, radius of transverse cross section and initial geometric imperfections, whose deviations are very important and sensitive for stability analysis to predict its critical pressure loading. Then six ring-stiffened ribs would be fitted to each elbow pipe and welded into elbow pipe together. Finally four elbow pipes with ring-stiffened ribs are welded together into the complete toroidal model. After the model is fabricated, the shape of the ring-stiffened toroidal model is checked again.

Here a ROMER machine is used to measure the shape of the model and analyze its imperfections, and such machine has an articulated arm which could obtain the absolute measurement of model’s size. The initial geometric imperfection of toroidal model in toroidal direction and cross-section direction can be measured respectively. For example Fig. 4 and Fig. 5 provide imperfections distribution shape at elbow D-1# along toroidal direction and at D7 half cross-section along circle direction.

Similar to the definition of cylindrical and spherical initial geometric imperfection and after measuring, the maximum local initial geometric imperfection \( f_0 \) of the toroidal shell is about 0.08t (t is the nominal average thickness of the toroidal shell) and the local maximum imperfection is bigger than the overall imperfection. After comparison the integrated initial geometric imperfection on toroidal model is shown in Table 2 (inward shell is positive). Here as same as the definition of ring-stiffened cylindrical shell, the local imperfection means the deflection of shell between two ribs in toroidal direction (\( \theta \)) or the region of \( \sqrt{\Delta \theta} \) in circle direction (\( \phi \)). And the overall imperfection means similarly the deflection of shell over segment or more than three ribs here in toroidal direction (\( \theta \)) or the region of \( \pi \Delta \phi / 2 \) in circle direction (\( \phi \)).

The measured thickness (t) varies from 9.6 mm to 11.2 mm. The measured thickness details of the toroidal model are shown in Table 3. And the average thickness of the toroidal shell is about 10.10 mm.

![Fig. 2. The model graphics and strain gauge locations.](image)

![Fig. 3. Manufacture process of elbow pipes accessory.](image)

![Fig. 4. Imperfections distribution shape at elbow D-1# along toroidal direction.](image)
2.2. Experiment of the toroidal model

The experiment of the toroidal model was carried out in a high pressure chamber whose configuration details are shown in Fig. 6 and its inside space size is $\phi 1500 \text{ mm} \times 3000 \text{ mm}$. Strain gauges are used to measure the strains at some points on this welding steel toroidal model under external pressure. The locations of the strain gauges should be on the key positions including the middle or end of two adjacent ribs on the outside or inside toroidal cycle where $\varphi = 0$ or $\pi$, such as points $A$, $A'$, $M$ and $N$ in Fig. 1. And the detailed locations of the strain gauges are shown in Fig. 2. The strain gauges are mainly set on elbow $D$ whose thickness is 10 mm the same as elbows $B$ and $C$, while the thickness of elbow $A$ is 12 mm. After strain gauges stuck on the model, the model is put into the pressure chamber (as shown in Fig. 7) and kept empty inside of model during the hydrostatic pressure test. This pressure test will be carried out through three main cycle steps, including residual stress elimination cycle, strain measurement cycle and final collapse test cycle. In the former two cycles, the test pressure will be kept at the design level in order that the model is still in static elastic stress state. While in the last cycle, the test pressure is gradually increased until the model collapses. This whole test step details are shown in Fig. 8. Here $p_{eq} = 20 \text{ MPa}$.

During the test process, the strains and loading pressure have been recorded timely in instrument and computer. Details are
provided in Zhang’s (2014) report. For example Fig. 9 shows the three direction strains on point M in Fig. 1 (on circle D7 in Fig. 2). From this figure, it can be easily found that the shell material on this key point had appeared obvious plastic flow before the test model is collapsed, and the plastic strain reached even more than 14000με before its collapse.

Fig. 10 shows the collapse deformation of the toroidal model after external hydrostatic pressure test. It can be found that the failure occurred on the outside surface of the toroidal model and it is the shell buckling failure mode for this model. And the test measured critical pressure loading is about 1.48 τr/a that is equal to 36.67 MPa.

3. Numerical analyses of the toroidal shell with ring-stiffened ribs

Because of geometric complexity and difficulty of theoretical solution to the toroidal shell with ring-stiffened ribs, the numerical method is generally used to investigate the structural performance of the toroidal shell with ribs. Especially to analyze the instability or buckling, the nonlinear FEM is adopted to analyze the toroidal shell with ring-stiffened ribs under uniform external pressure. The nonlinear structural characteristics of the toroidal shell with ribs will be discussed.

3.1. Nonlinear numerical method

The pressure shell under external pressure can have large deformations and/or large rotations and thus enter into a post-buckling phase. There is also the likelihood of some material entering a plastic state, depending on the elastic-plastic properties of the material. Therefore, the nonlinear FE method is a good means to simulate and analyze the buckling of the toroidal shell with ring-stiffened ribs subjected to external hydrostatic pressure and even to reveal its characteristics after entering into a post-buckled phase.

Buckling analysis should include not only large deformation and nonlinear strain-displacement relations, but also material...
nonlinearity. In this paper, the measured stress–strain curve is adopted, see Fig. 11, which comes from the standard specimen test supplied by the manufacturer.

A key question here is how to judge when the shell element enters into yield level and the toroidal structure would be buckling or even whole toroidal shell model enters into a post-buckling plasticity flow phase. Here the Von-Mises yield criterion has been adopted to judge whether the toroidal structure shell has entered into the yielding state

\[ F(J_2) = \sigma_{eq} - \sigma_t = \sqrt{\frac{3}{2}S_{ij}S_{ij}} - \sigma_t = 0 \]  

Generally the equivalent Von-Mises stress \( \sigma_{eq} \) could be expressed as follows:

\[ \sigma_{eq} = \sqrt{3J_2} \]  

\[ J_2 = \frac{1}{2}S_{ij}S_{ij} = \frac{1}{6}[(\sigma_{XX} - \sigma_{YY})^2 + (\sigma_{YY} - \sigma_{ZZ})^2 + (\sigma_{ZZ} - \sigma_{XX})^2] + \sigma_{XY}^2 + \sigma_{YZ}^2 + \sigma_{ZX}^2 \]  

where \( S_{ij} \) is the deviatoric tensor of stress; \( \sigma_t \) is the yield stress; \( \sigma_{XX}, \sigma_{YY}, \sigma_{ZZ} \) are the normal stresses in the X, Y and Z directions; \( \sigma_{XY}, \sigma_{YZ}, \sigma_{ZX} \) are the shear stresses.

In the presented nonlinear FEM, the toroidal model is first built and then the linear-elastic and eigenvalue buckling analysis is conducted. If the real initial geometric imperfections have been measured, then these would be added to the toroidal model. Otherwise the first mode of eigenvalues buckling will be used to simulate the geometric imperfections, with the amplitude being determined by the manufacturing quality level. To solve the nonlinear differential equations, both Newton method and arc-length control method can be used in this numerical solution method. However the arc-length control method is more convenient to obtain the post-buckling solution for each iteration step. Based on the nonlinear FEA calculations, the critical buckling pressure loading and other results such as deformation-pressure curve and stress or strain contours can also be obtained.

3.2. Nonlinear FE analyses of the toroidal shell with ring-stiffened ribs

The structural response is generally determined by its shape, external loading, and the initial geometric imperfections and welding residual stresses. For the toroidal shell with ring-stiffened ribs subjected to uniform external pressure loading, its structural shape and initial geometric imperfection will mostly decide its nonlinear structural characteristics.

The nonlinear FE analyses are carried out to study the buckling behavior of the toroidal shell with ring-stiffened ribs under uniform external pressure. Its ultimate pressure loading and final collapsed/buckling failure shape can also be obtained through these analyses.

In author’s previous work Du et al. (2010) about the ring-stiffened toroidal shell, similar to ring-stiffened cylindrical shell, a toroidal section model had been adopted to analyze its structural characteristics, which has verified that the ring-stiffened toroidal shell is super to the ring-stiffened cylinder. Meanwhile it can be seen in Fig. 10 that the collapsed region appears in an elbow pipe scope. Therefore a toroidal section model with some special parameters, which is just as a cabin, has been used to solve and analyze its ultimate critical state and then compare with model experimental results. Its main parameters are shown in Table 1 but
here only one elbow pipe with ring-stiffened ribs is used for analysis, because the final collapse happened in one elbow from experimental results in Fig. 10. Here it is assumed that the model is subjected to uniform external pressure and it has two symmetrical boundaries on both ends of the model.

Fig. 12 shows the deformation contour of the toroidal model segments corresponding to the critical pressure loading and entering into plastic buckling flow. Based on the thin-shell theory, it can be easily found that the shell buckling failure generally appears on the outside circle of the toroidal shell where Gaussian curvature ratio is positive.

Fig. 13 shows two non-dimensional pressure–deflection curves, one collapsed after critical pressure and another well into an ideal post-bucking state. From this figure, a sharp drop in critical pressure is noticed at the critical stage followed by further dropping through the post-buckled plastic flow, where it is assumed that the model material has good elastic plastic property. The result in this figure shows the local pressure-flexibility curve up to the critical state. It can be seen that the curve is almost linear till the critical limit but changes abruptly to fall into the plastic flow phase. While Fig. 14 shows the influence of the initial geometric deflection $f_0$ on the critical pressure of the ring-stiffened toroidal model and it can be found that the error between test and FEA is about 6%.

At the same time it should be noticed that the model test had been stopped when test pressure reached $1.2\tau_{0s}/\alpha$ and pressure–strain curve (in Fig. 9) was measured at this time. And then after being laid aside for long time, the toroidal model was set into pressure chamber again and finished its collapsed test. It is thought that the material may have appeared plastic intensifying which causes the test result higher than FEA.

3.3. Comparison and verification of buckling analysis with experiment model

In summary FE analyses can be easily carried out to obtain the buckling pressure of the toroidal shell with ring-stiffened ribs by different parameters, while experiment model is generally only
built in some special parameters and demonstrates its certain structural characteristic under external uniform pressure loading. As a matter of the fact, these two methods could be compared and checked by each other. Therefore the whole toroidal model should be solved by FE method and used to study the structural behavior of the toroidal model thoroughly.

Based on the model parameters shown in Table 1, finite element model can be generated as Fig. 15 and the initial geometric imperfections measured from the real test model, shown in Table 2, have also been added to the finite element model here. Then after the buckling calculation by nonlinear FE method, its critical pressure or collapse deformation has been obtained completely.

Generally the membrane stress or total stress strength could be expressed as follows:

\[
\sigma_m = k_m \frac{p a}{t}, \quad \sigma_{tr} = k_{tr} \frac{p a}{t}
\]  

(4)

where \(k_m\) and \(k_{tr}\) are membrane stress strength coefficient and total stress strength coefficient.

By the FE method, the stress status can be obtained after static elastic calculation. The membrane stress strength coefficient \(k_m\) or total stress strength coefficient \(k_{tr}\) are shown respectively in Fig. 16 and Fig. 17. From these two figures, it can be obviously found that the maximum \(k_m\) is near to 1 and minimum \(k_m\) is near to 2/3 while the maximum \(k_{tr}\) is equal to 1.25 that is located in negative Gaussian ratio shell. But in fact the nominal stress is \(p a/t\) which represents the maximum membrane stress in circumferential direction on cylinder subjected to uniform external pressure \(p\).

By the nonlinear FE method, the tested model has been solved to obtain its critical pressure and collapsed deformation. Fig. 18 shows the deformation contour of ring-stiffened toroidal model under the critical pressure. From this figure, it can be seen that the buckling firstly appeared on the toroidal shell that locates on the outside surface of the toroidal model. In addition, half of ring-stiffened toroidal model has also been calculated by nonlinear FE method and the collapsed deformation is shown in Fig. 19. And it is easily noticed that the collapsed position and collapsed region shape in half model are almost similar to those of whole ring-stiffened toroidal model calculation, which is also similar as elbow’s result in Fig. 12. Meanwhile by comparing Fig. 10 with Figs. 18 and 19, one can obviously find that the numerical failure mode is in agreement with the experimental failure model.

The predicted critical pressure from the whole model is about 1.38\(\sigma_s/a\) that is the same as the result from toroidal elbow model. While compared to the experimental measured critical pressure, the error between them is only about 6% which confirms that the nonlinear FE analyses can be used to study the structural behavior of the ring-stiffened toroidal pressure shell.

Meanwhile the plain toroidal model without ribs but with parameters from Table 1 and another thicker toroidal model in which the reinforcing ribs are converted into the additional wall thickness have also been solved by presented nonlinear FEM. their critical pressures are respectively about 0.91\(\sigma_s/a\) and 1.18\(\sigma_s/a\) while experimental result is 1.48\(\sigma_s/a\). Therefore it can be obviously noticed that the ring-stiffened ribs on this toroidal model have improved its critical pressure.
4. Parametric study on the structural behavior of the toroidal pressure shell

As is well-known the cylindrical or spherical shell under external pressure may fail by either strength collapse or instability/buckling failure. The ring-stiffened toroidal shell, a new type of the pressure hulls in underwater engineering, may also have different failure modes for different structural parameters such as the size of toroidal shell or ring-stiffened ribs. This problem is investigated as follows.

4.1. Strength failure of ring-stiffened toroidal shell

To the negative Gaussian curve ratio part of the toroidal shell, the toroidal shell is subjected to uniform pressure such as a vessel shell under internal pressure loading. Then the strength failure will mainly occur on this part and material on the shell may enter into plastic state until damage. It can be easily found from Fig. 16 and Fig. 17 that the membrane stress strength and total stress strength are both maximum in such region. Therefore according to author’s works Du et al. (2010) and pressure vessel rule (ASME VIII-2, 2007), the toroidal shell will appear strength damage if the critical pressure \( p_{cr} \) satisfy the following formula:

\[
p_{cr} = \frac{3n_3(R-a)}{a} \left( \frac{2R-a}{2K} \right) \]

Fig. 20 shows the critical strength failure line varied by the main toroidal parameter \( R/a \) and the tested toroidal model result. From this figure, it can be obviously found that the critical pressure of the tested toroidal model is lower than that strength failure pressure. Therefore this confirmed that the failure mode of the tested model is not the strength breaking, which has also been verified by experimental results (see Fig. 10).

4.2. Elastic buckling analysis of ring-stiffened toroidal shell

However on the other side/part of the toroidal shell with positive Gaussian curve ratio, the toroidal shell is typically subjected to uniform external pressure. When the ribs are strong enough and meanwhile the shell thick enough to resist pressure strength on the side with negative Gaussian curve ratio, the failure collapse would happen firstly as the toroidal shell buckling. Otherwise the ribs will cause the structural hull buckling firstly.

Referring to the investigation of cylindrical shell with ring-stiffened ribs or spherical shell under external uniform pressure, it is the most important question to obtain elastic buckling modes of ring-stiffened toroidal shell subjected to external uniform pressure.
capability to buckling instability including ribs collapse or shell buckling between two adjacent ring-stiffened ribs. However the initial geometric imperfection including out-of-roundness would affect directly on structural stability of resisting external pressure, especially for the critical loading of pressure hull under external pressure. The influence of initial geometric imperfection on the critical pressure loading of ring-stiffened toroidal model can be easily found from Figure 14, which indicates that the critical pressure will decrease sharply as the initial deflection arising. Therefore it is necessary to study the structural behavior of the toroidal pressure hull under external pressure, including its structural and failure property.

The initial geometric imperfection can be measured exactly after the model manufacture. Simultaneously its shape and amplitude is obviously irregular and random in any structural hull. And for the conservative consideration in Section 3, the measured data of initial local geometric imperfection was added onto the first appearing mode of eigenvalue buckling of ring-stiffened toroidal model, which is closely near to the shape feature of initial geometric imperfection and it is second-order mode of shell buckling.

Therefore the geometric imperfection of an arbitrary point on tested model may be assumed that it can be asymptotically expressed and described by its natural elastic buckling modes as follows:

\[ f_0 = \sum_{i=1}^{N} a_i \phi_i \]  

where \( a_i \) is the arbitrary coefficient and \( \phi_i \) is the amplitude of eigen-mode \( i \). Therefore in Sections 3, \( N = 2 \) and \( a_1 = 0 \).

The experimental results in Figure 10 and numerical results in Figure 18 or Figure 19 indicate that the initial geometric imperfection is important to the critical pressure, collapse shape and failure mode. If the initial imperfection just includes the overall imperfection on the model, it will appear the ribs buckling mode as Figure 21(a).

For the first failure mode, the stiffness of ring-stiffened ribs is the primary parameter which mainly determines its collapse loading and its failure shape. Therefore a coefficient \( \zeta \) which means the stiffness of ring-stiffened ribs to the shell stiffness is defined as follows:

\[ \zeta = \frac{E I_z}{2 \pi D} \]  

\[ D = \frac{E t^3}{12(1-\mu^2)} \]

where \( I_z \) is the second moment of rib’s cross-section about its center. \( E \) and \( \mu \) are respectively Young’s modulus and Poisson’s ratio.

By calculation of presented nonlinear FE method and changing size of ring-stiffened ribs of experimental model to achieve different stiffness of ribs, the structural failure categories of ring-stiffened toroidal shell could be obtained and shown in Table 4. The main parameters except parameters of ribs are all the same as

---

**Table 4**

<table>
<thead>
<tr>
<th>( \zeta )</th>
<th>( P_{\text{cr}}(\sigma_{\text{st}}) )</th>
<th>Failure categories</th>
<th>( \zeta )</th>
<th>( P_{\text{cr}}(\sigma_{\text{st}}) )</th>
<th>Failure categories</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.23611</td>
<td>13.4</td>
<td>Ribs buckling</td>
<td>0.97359</td>
<td>22.69</td>
<td>Shell buckling</td>
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<tr>
<td>0.2904</td>
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<td>0.104691</td>
<td>22.68</td>
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<td>1.11668</td>
<td>22.68</td>
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<tr>
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<tr>
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</table>

**Fig. 23.** The elastic buckling pressure of the ring-stiffened toroidal model verifying by \( \zeta \).
Table 1. These data are plotted as shown in Fig. 23. From Table 4 and Fig. 23, it can be easily noticed that when $\zeta$ is larger than 0.9, the shell buckling will appear firstly and otherwise the ribs instability would occur preferentially.

It should be noticed that although the first failure natural mode of ring-stiffened toroidal shell is general ribs buckling, the local initial geometric real imperfection would lead to the model appearing shell buckling finally.

5. Summary

In this paper, a steel welding ring-stiffened toroidal model is tested successfully in pressure chamber. Model experimental results achieved the critical pressure loading and collapse shape. Then, nonlinear FE analyses are carried out to study the structural characteristics of the tested toroidal model and confirmed by experimental results. Furthermore the essential causation and collapse mode of tested toroidal model have been studied by the nonlinear FE method. The experimental and numerical investigation reveal that initial geometric imperfection would determine the model’s final failure mode, critical pressure and collapse deformation. Moreover the buckling failure modes, shape and even critical pressure of ring-stiffened toroidal model are basically the same in FE analysis and experiment. Therefore the nonlinear numerical FE method can be used as a reliable tool to obtain critical pressure solution of toroidal shell with ring-stiffened ribs.

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